

Strong coupling constants of light pseudoscalar mesons with heavy baryons in QCD

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Abstract

We calculate the strong coupling constants of light pseudoscalar mesons with heavy baryons within the light cone QCD sum rules method. It is shown that sextet–sextet, sextet–antitriplet and antitriplet–antitriplet transitions are described by one universal invariant function for each class. A comparison of our results on the coupling constants with the predictions existing in literature is also presented.

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1 Introduction

In this decade exciting experimental results have been obtained in heavy baryon spectroscopy. During these years, the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ antitriplet states, Λ_c^+ , Ξ_c^+ , Ξ_c^0 and $\Lambda_c^+(2593)$, $\Xi_c^+(2790)$, $\Xi_c^0(2790)$ and the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ and sextet states, Ω_c^* , Σ_c^* , Ξ_c^* have been observed in experiments [1]. Among the s-wave bottom hadrons, only Λ_b , Σ_b , Σ_b^* , Ξ_b and Ω_b have been discovered. Moreover, in recent years many new states have been observed by BaBar and BELLE collaborations, such as, $X(3872)$, $Y(3930)$, $Z(3930)$, $X(3940)$, $Y(4008)$, $Z_1^+(4050)$, $Y(4140)$, $X(4160)$, $Z_2(4250)$, $Y(4260)$, $Y(4360)$, $Z^+(4430)$, and $Y(4660)$ which remain unidentified.

Of course, establishing these states is a remarkable progress in hadron physics. It is expected that LHC, the world's largest-highest-energy particle accelerator, will open new horizons in the discovery of the excited bottom baryon states [2]. The experimental progress on heavy hadron spectroscopy stimulated intensive theoretical studies in this respect (for a review see [3,4] and references therein). A detailed theoretical study of experimental results on hadron spectroscopy and various weak and strong decays can provide us with useful information about the quark structure of new hadrons at the hadronic scale.

This scale belongs to the nonperturbative sector of QCD. Therefore, for calculation of the form factors in weak decays and coupling constants in strong decays, some nonperturbative methods are needed. Among many nonperturbative methods, QCD sum rules [5] is more reliable and predictive. In the present work, we calculate the strong coupling constants of light pseudoscalar mesons with sextet and antitriplet baryons, in light cone version of the QCD sum rules (LCSR) method (for a review, see [6]). Note that some of the strong coupling constants have already been studied in [7–9] in the same framework.

The outline of this article is as follows. In section 2, we demonstrate how coupling constants of pseudoscalar mesons with heavy baryons can be calculated. In this section, the LCSR for the heavy baryon–pseudoscalar meson coupling constants are also derived using the most general form of the baryon currents. Section 3 is devoted to the numerical analysis and a comparison of our results with the existing predictions in the literature.

2 Light cone QCD sum rules for the coupling constants of pseudoscalar mesons with heavy baryons

Before presenting the detailed calculations for the strong coupling constants of pseudoscalar mesons with heavy baryons, we would like to make few remarks about the classification of heavy baryons. Heavy baryons with a single heavy quark belong to either $SU(3)$ antisymmetric $\bar{3}_F$ or symmetric 6_F flavor representations. Since we consider the ground states, the total spin of the two light quarks must one for 6_F and zero for $\bar{3}_F$, due to the symmetry property of their colors and flavors, as a result of which we can write $J^P = \frac{1}{2}^+/\frac{3}{2}^+$ for 6_F and $J^P = \frac{1}{2}^+$ for $\bar{3}_F$. Graphically, 6_F and $\bar{3}_F$ representations are given in Fig. (1), where α , $\alpha+1$, $\alpha+2$ determine the charges of baryons ($\alpha = -1$ or 0), and the asterisk (*) denote $J^P = \frac{3}{2}^+$ states. In this work, we will consider only $J^P = \frac{1}{2}^+$ states.

After this preliminary remarks, we proceed by calculating the strong coupling constants of pseudoscalar mesons with heavy baryons within the LCSR. For this purpose, we start

by considering the following correlation function:

$$\Pi^{(ij)} = i \int d^4x e^{ipx} \langle \mathcal{P}(q) | \mathcal{T} \{ \eta^{(i)}(x) \bar{\eta}^{(j)}(0) \} | 0 \rangle , \quad (1)$$

where $\mathcal{P}(q)$ is the pseudoscalar-meson with momentum q , η is the interpolating current for the heavy baryons and \mathcal{T} is the time ordering operator. Here, $i = 1, j = 1$ describes the sextet-sextet, $i = 1, j = 2$ corresponds to sextet-triplet, and $i = 2, j = 2$ describes triplet-triplet transitions. For convenience we shall denote $\Pi^{(11)} = \Pi^{(1)}$, $\Pi^{(12)} = \Pi^{(2)}$ and $\Pi^{(22)} = \Pi^{(3)}$. The sum rules for the coupling constants of pseudoscalar mesons with heavy baryons can be obtained by calculating the correlation function (1) in two different ways, namely, in terms of the hadrons and in terms of quark gluon degrees of freedom, and then matching these two representations.

Firstly, we calculate the correlation function (1) in terms of hadrons. Inserting complete sets of hadrons with the same quantum numbers in the interpolating currents and isolating the ground states, we obtain

$$\Pi^{(ij)} = \frac{\langle 0 | \eta^{(i)}(0) | B_2(p) \rangle \langle B_2(p) \mathcal{P}(q) | B_1(p+q) \rangle \langle B_1(p+q) | \bar{\eta}^{(j)}(0) | 0 \rangle}{(p^2 - m_2^2) [(p+q)^2 - m_1^2]} + \dots , \quad (2)$$

where $|B_2(p)\rangle$ and $|B_1(p+q)\rangle$ are the $\frac{1}{2}$ states, and m_2 and m_1 are their masses, respectively. The dots in Eq. (2) describe contributions of the higher states and continuum. It follows from Eq. (2) that in order to calculate the correlation function in terms of hadronic parameters, the matrix elements entering to Eq. (2) are needed. These matrix elements are defined in the following way:

$$\begin{aligned} \langle 0 | \eta^{(i)} | B(p) \rangle &= \lambda_i u(p) , \\ \langle B(p+q) | \eta^{(j)} | 0 \rangle &= \lambda_j \bar{u}(p+q) , \\ \langle B(p) \mathcal{P}(q) | B(p+q) \rangle &= g \bar{u}(p) i \gamma_5 u(p+q) , \end{aligned} \quad (3)$$

where λ_i and λ_j are the residues of the heavy baryons, g is the coupling constant of pseudoscalar meson with heavy baryon and u is the Dirac bispinor.

Using Eqs. (2) and (3) and performing summation over spins of the baryons, we obtain the following representation of the correlation function from the hadronic side:

$$\Pi^{(ij)} = i \frac{\lambda_i \lambda_j g}{(p^2 - m_2^2) [(p+q)^2 - m_1^2]} \{ \not{p} \gamma_5 + \text{other structures} \} , \quad (4)$$

where we kept the structure which leads to a more reliable result.

In order to calculate the correlation function from QCD side, the forms of the interpolating currents for the heavy baryons are needed. The general form of the interpolating currents for the heavy spin $\frac{1}{2}$ sextet and antitriplet baryons can be written as (see for example [10]),

$$\begin{aligned} \eta_Q^{(s)} &= -\frac{1}{\sqrt{2}} \epsilon^{abc} \left\{ \left(q_1^{aT} C Q^b \right) \gamma_5 q_2^c + \beta \left(q_1^{aT} C \gamma_5 Q^b \right) q_2^c - \left[\left(Q^{aT} C q_2^b \right) \gamma_5 q_1^c + \beta \left(Q^{aT} C \gamma_5 q_2^b \right) q_1^c \right] \right\} , \\ \eta_Q^{(anti-t)} &= \frac{1}{\sqrt{6}} \epsilon^{abc} \left\{ 2 \left(q_1^{aT} C q_2^b \right) \gamma_5 Q^c + 2\beta \left(q_1^{aT} C \gamma_5 q_2^b \right) Q^c + \left(q_1^{aT} C Q^b \right) \gamma_5 q_2^c + \beta \left(q_1^{aT} C \gamma_5 Q^b \right) q_2^c \right. \\ &\quad \left. + \left(Q^{aT} C q_2^b \right) \gamma_5 q_1^c + \beta \left(Q^{aT} C \gamma_5 q_2^b \right) q_1^c \right\} , \end{aligned} \quad (5)$$

where a, b, c are the color indices and β is an arbitrary parameter. It should also be noted that the general form of interpolating currents for light spin 1/2 baryons was introduced in [11] and $\beta = -1$ corresponds to the Ioffe current [12]. The quark fields q_1 and q_2 for the sextet and antitriplet are presented in Table 1.

	$\Sigma_{b(c)}^{+(++)}$	$\Sigma_{b(c)}^{0(+)}$	$\Sigma_{b(c)}^{- (0)}$	$\Xi_{b(c)}^{- (0) '}$	$\Xi_{b(c)}^{0(+) '}$	$\Omega_{b(c)}^{- (0)}$	$\Lambda_{b(c)}^{0(+)}$	$\Xi_{b(c)}^{- (0)}$	$\Xi_{b(c)}^{0(+)}$
q_1	u	u	d	d	u	s	u	d	u
q_2	u	d	d	s	s	s	d	s	s

Table 1: The quark flavors q_1 and q_2 for the baryons in the sextet and the antitriplet representations

As has already been noted, in order to calculate the coupling constants of pseudoscalar mesons with heavy baryons entering to sextet and antitriplet representation, the calculation of the correlation function from QCD part is needed. Before calculating it, we follow the approach given in [13–17] and try to find relations among invariant functions involving coupling constants of pseudoscalar mesons with sextet and antitriplet baryons. We will show that the correlation functions responsible for coupling constants of pseudoscalar mesons (P) with sextet–sextet (SS), sextet–antitriplet (SA) and antitriplet–antitriplet (AA) baryons can each be represented in terms of only one invariant function. Of course, the form of the invariant functions for the couplings SSP, SAP and AAP are different in the general case. It should be noted here that the relations presented below are all structure independent.

We start our discussion by considering the sextet–sextet transition, concretely. Consider the $\Sigma_b^0 \rightarrow \Sigma_b^0 \pi^0$ transition. The invariant function for this transformation can be written in the following form

$$\Pi^{\Sigma_b^0 \rightarrow \Sigma_b^0 \pi^0} = g_{\pi \bar{u}u} \Pi_1^{(1)}(u, d, b) + g_{\pi \bar{d}d} \Pi_1^{'(1)}(u, d, b) + g_{\pi \bar{b}b} \Pi_2^{(1)}(u, d, b) , \quad (6)$$

where the interpolating current of π^0 meson is written as

$$J_{\pi^0} = \sum_{u,d} g_{\pi \bar{q}q} \bar{q} \gamma_5 q .$$

Obviously, the relations $g_{\pi \bar{u}u} = -g_{\pi \bar{d}d} = \frac{1}{\sqrt{2}}$, $g_{\pi \bar{b}b} = 0$ hold for the π^0 meson. The invariant functions Π_1 , Π_1' and Π_2 describe the radiation of π^0 meson from u , d and b quarks of Σ_b^0 baryon, respectively, and they can formally be defined as:

$$\begin{aligned} \Pi_1^{(1)}(u, d, b) &= \langle \bar{u}u | \Sigma_b^0 \bar{\Sigma}_b^0 | 0 \rangle , \\ \Pi_1^{'(1)}(u, d, b) &= \langle \bar{d}d | \Sigma_b^0 \bar{\Sigma}_b^0 | 0 \rangle , \\ \Pi_2^{(1)}(u, d, b) &= \langle \bar{b}b | \Sigma_b^0 \bar{\Sigma}_b^0 | 0 \rangle . \end{aligned} \quad (7)$$

It follows from the definition of the interpolating current of Σ_b baryon that it is symmetric under the exchange $u \leftrightarrow d$, hence $\Pi_1^{'(1)}(u, d, b) = \Pi_1^{(1)}(d, u, b)$. Using this relation, we immediately get from Eq. (6) that,

$$\Pi^{\Sigma_b^0 \rightarrow \Sigma_b^0 \pi^0} = \frac{1}{\sqrt{2}} \left[\Pi_1^{(1)}(u, d, b) - \Pi_1^{(1)}(d, u, b) \right] , \quad (8)$$

and one can easily see that in the $SU_2(2)_f$ limit, $\Pi^{\Sigma_b^0 \rightarrow \Sigma_b^0 \pi^0} = 0$.

The invariant function responsible for the $\Sigma_b^+ \rightarrow \Sigma_b^+ \pi^0$ transition can be obtained from the $\Sigma_b^0 \rightarrow \Sigma_b^0 \pi^0$ case by making the replacement $d \rightarrow u$, and using $\Sigma_b^0 = -\sqrt{2}\Sigma_b^+$, from which we get,

$$4\Pi_1^{(1)}(u, d, b) = -2 \langle \bar{u}u | \Sigma_b^+ \bar{\Sigma}_b^+ | 0 \rangle . \quad (9)$$

Appearance of the factor 4 on the left hand side is due to the fact that each Σ_b^+ contains two u quark, hence there are 4 possible ways for radiating π^0 from the u quark. Making use of Eq.(8), we get

$$\Pi^{\Sigma_b^+ \rightarrow \Sigma_b^+ \pi^0} = \sqrt{2}\Pi_1^{(1)}(u, u, b). \quad (10)$$

The invariant function describing $\Sigma_b^- \rightarrow \Sigma_b^- \pi^0$ can easily be obtained from the $\Sigma_b^0 \rightarrow \Sigma_b^0 \pi^0$ transition by making the replacement $u \rightarrow d$ and taking into account $\Sigma_b^0(u \rightarrow d) = \sqrt{2}\Sigma_b^-$. Performing calculation similar to the previous case, we get

$$\Pi^{\Sigma_b^- \rightarrow \Sigma_b^- \pi^0} = \sqrt{2}\Pi_1^{(1)}(d, d, b). \quad (11)$$

Now, let us proceed to obtain the results for the invariant function involving $\Xi_b'^{(0)-} \rightarrow \Xi_b'^{(0)-} \pi^0$ transition. The invariant function for this transition can be obtained from the $\Sigma_b^0 \rightarrow \Sigma_b^0 \pi^0$ case using the fact that $\Xi_b'^0 = \Sigma_b^0(d \rightarrow s)$ and $\Xi_b'^- = \Sigma_b^0(u \rightarrow s)$. As a result, we obtain

$$\begin{aligned} \Pi^{\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0} &= \frac{1}{\sqrt{2}}\Pi_1^{(1)}(u, s, b) , \\ \Pi^{\Xi_b'^- \rightarrow \Xi_b'^- \pi^0} &= -\frac{1}{\sqrt{2}}\Pi_1^{(1)}(d, s, b) . \end{aligned} \quad (12)$$

Obtaining relations among the invariant functions involving charged π^\pm mesons requires more care. In this respect, we start by considering the matrix element $\langle \bar{d}d | \Sigma_b^0 \bar{\Sigma}_b^0 | 0 \rangle$, where d quarks from the Σ_b^0 and $\bar{\Sigma}_b^0$ from the final $\bar{d}d$ state, and u and b quarks are the spectators. The matrix element $\langle \bar{u}d | \Sigma_b^+ \bar{\Sigma}_b^0 | 0 \rangle$ describes the case where d quark from $\bar{\Sigma}_b^0$ and u quark from Σ_b^+ form the $\bar{u}d$ state and the remaining u and b are being again the spectators. One can expect from this observation that these matrix elements should be proportional to each other and calculations confirm this expectation. So,

$$\Pi^{\Sigma_b^0 \rightarrow \Sigma_b^+ \pi^-} = \langle \bar{u}d | \Sigma_b^+ \bar{\Sigma}_b^0 | 0 \rangle = -\sqrt{2} \langle \bar{d}d | \Sigma_b^0 \bar{\Sigma}_b^0 | 0 \rangle = -\sqrt{2}\Pi_1^{(1)}(d, u, b) . \quad (13)$$

Making the replacement $u \leftrightarrow d$ in Eq. (13), we obtain

$$\Pi^{\Sigma_b^0 \rightarrow \Sigma_b^- \pi^+} = \langle \bar{d}u | \Sigma_b^- \bar{\Sigma}_b^0 | 0 \rangle = \sqrt{2} \langle \bar{u}u | \Sigma_b^0 \bar{\Sigma}_b^0 | 0 \rangle = \sqrt{2}\Pi_1^{(1)}(u, d, b) . \quad (14)$$

In estimating the coupling constants of SSP, SAP and AAP, it is enough to consider the $\Sigma_b^0 \rightarrow \Sigma_b^0 P$, $\Xi_b'^0 \rightarrow \Xi_b'^0 P$ and $\Xi_b'^- \rightarrow \Xi_b'^- P$ transitions, respectively. All remaining transitions can be obtained from these transitions with the help of the appropriate transformations among quark fields. Relations among the invariant functions of the charmed baryons can

easily be obtained by making the replacement $b \rightarrow c$ and adding to charge of each baryon a positive unit charge.

Performing similar calculations, one can obtain rest of the required expressions from the correlation functions in terms of the invariant function Π_1 , involving π , K and η mesons describing sextet–sextet, sextet–antitriplet and antitriplet–antitriplet transitions. In the present work, we neglect the mixing between η and η' mesons. It should also be noted here that all coupling constants for the SSP, SAP and AAP are described only by one invariant function in each class of transitions, but the forms of the invariant functions in each group of transitions are different.

The invariant function Π_1 responsible for the $\Sigma_b^0 \rightarrow \Sigma_b^0 P$, $\Xi_b'^0 \rightarrow \Xi_b^0 P$ and $\Xi_b^0 \rightarrow \Xi_b^0 P$ transitions can be calculated in deep Euclidean region, $-p^2 \rightarrow +\infty$ and $-(p+q)^2 \rightarrow +\infty$ using the operator product expansion (OPE) in terms of the distribution amplitudes (DA's) of the pseudoscalar mesons and light and heavy quark operators. Up to twist-4 accuracy, the matrix elements $\langle P(q) | \bar{q}(x) \Gamma q(0) | 0 \rangle$ and $\langle P(q) | \bar{q}(x) G_{\mu\nu} q(0) | 0 \rangle$, where Γ is any arbitrary Dirac matrix, are determined in terms of the DA's of the pseudoscalar mesons, and their explicit expressions are given in [18–20].

The light and heavy quark propagators are calculated in [21], and [22], respectively. Using expressions of these propagators and definitions of DA's for the pseudoscalar mesons, the correlation function can be calculated from the QCD side, straightforwardly. Equating the coefficients of the structure $\not{p}\gamma_5$ of the representation of the correlation function from hadronic and theoretical sides, and applying the Borel transformation with respect to the variables p^2 and $(p+q)^2$ in order to suppress the contributions of the higher states and continuum, we obtain the following sum rules for the strong coupling constants of the pseudoscalar mesons with sextet and antitriplet baryons:

$$g^{(i)} = \frac{1}{\lambda_1^{(i)} \lambda_2^{(i)}} e^{\frac{m_1^{(i)2}}{M_1^2} + \frac{m_2^{(i)2}}{M_2^2}} \Pi_1^{(i)}, \quad (15)$$

where, $i = 1, 2$ and 3 for sextet–sextet, sextet–antitriplet and antitriplet–antitriplet, respectively and M_1^2 and M_2^2 are the Borel masses corresponding to the initial and the final baryons. Since the masses of the initial and final baryons are practically equal to each other, we take $M_1^2 = M_2^2 = 2M^2$; and $\lambda_1^{(i)}$ and $\lambda_2^{(i)}$ are the residues of the initial and final baryons, respectively, which are calculated in [23]. The explicit expressions for $\Pi_1^{(i)}$ are quite lengthy and we do not present all of them here. As an example, we only present the explicit expression of the $\Pi_1^{(1)}$, which is given as:

$$\begin{aligned} e^{m_Q^2/M^2 - m_P^2/M^2} \Pi_1^{(1)}(u, d, b) = & \frac{(1-\beta)^2}{32\sqrt{2}\pi^2} M^4 m_Q^3 f_P \left[I_2 - m_Q^2 I_3 \right] \phi_\eta(u_0) + \frac{(1-\beta^2)}{64\sqrt{2}\pi^2} M^4 m_Q^2 \mu_P \left\{ \left(i_3(\mathcal{T}, 1) - 2i_3(\mathcal{T}, v) \right) I_2 \right. \\ & - 2m_Q^2 \left[i_3(\mathcal{T}, 1) - 2i_3(\mathcal{T}, v) + (1 - \tilde{\mu}_P^2) \phi_\sigma(u_0) \right] I_3 \Big\} \\ & - \frac{(1-\beta)^2}{128\sqrt{2}\pi^2} M^2 m_P^2 m_Q f_P \left\{ m_Q^2 \mathbb{A}(u_0) I_2 - 2 \left(i_2(\mathcal{V}_\parallel, 1) - 2i_2(\mathcal{V}_\perp, 1) \right) I_1 \right. \\ & \left. - 2m_Q^2 \left[i_2(\mathcal{A}_\parallel, 1) - 2 \left(i_2(\mathcal{V}_\parallel, 1) - i_2(\mathcal{V}_\perp, 1) + i_2(\mathcal{A}_\parallel, v) \right) \right] I_2 \right\} \end{aligned}$$

$$\begin{aligned}
& - \frac{(1-\beta^2)}{96\sqrt{2}\pi^2} M^2 \left[3m_{\mathcal{P}}^2 m_Q^2 \mu_{\mathcal{P}} \left(2m_Q^2 I_3 - I_2 \right) \left(i_2(\mathcal{T}, 1) - 2i_2(\mathcal{T}, v) \right) - 4\langle \bar{d}d \rangle f_{\mathcal{P}} \pi^2 \phi_{\eta}(u_0) \right] \\
& + \frac{(1-\beta^2)}{384\sqrt{2}M^6} m_{\mathcal{P}}^2 m_Q^4 m_0^2 f_{\mathcal{P}} \langle \bar{d}d \rangle \mathbb{A}(u_0) \\
& - \frac{1}{2304\sqrt{2}M^4} m_Q^2 m_0^2 \langle \bar{d}d \rangle \left\{ (1-\beta^2) m_{\mathcal{P}}^2 f_{\mathcal{P}} \left[5\mathbb{A}(u_0) + 12 \left(i_2(\mathcal{A}_{\parallel}, 1) + i_2(\mathcal{V}_{\parallel}, 1) - 2i_2(\mathcal{A}_{\parallel}, v) \right) \right] \right. \\
& \left. - 8m_Q \mu_{\mathcal{P}} (1 - \tilde{\mu}_{\mathcal{P}}^2) [3 + \beta(2 + 3\beta)] \phi_{\sigma}(u_0) \right\} \\
& - \frac{1}{864\sqrt{2}M^2} m_Q \langle \bar{d}d \rangle \left\{ 9(1-\beta^2) m_Q f_{\mathcal{P}} \left(m_{\mathcal{P}}^2 \mathbb{A}(u_0) + m_0^2 \phi_{\eta}(u_0) \right) + 2[5 + \beta(4 + 5\beta)] m_0^2 \mu_{\mathcal{P}} (1 - \tilde{\mu}_{\mathcal{P}}^2) \phi_{\sigma}(u_0) \right\} \\
& - \frac{1}{576\sqrt{2}} \left\{ (1-\beta^2) f_{\mathcal{P}} \langle \bar{d}d \rangle \left[6m_{\mathcal{P}}^2 \mathbb{A}(u_0) - 12m_{\mathcal{P}}^2 \left(i_2(\mathcal{A}_{\parallel}, 1) + i_2(\mathcal{V}_{\parallel}, 1) - 2i_2(\mathcal{A}_{\parallel}, v) \right) + m_0^2 \phi_{\eta}(u_0) \right] \right. \\
& \left. + 8[3 + \beta(2 + 3\beta)] m_Q \mu_{\mathcal{P}} (1 - \tilde{\mu}_{\mathcal{P}}^2) \langle \bar{d}d \rangle \phi_{\sigma}(u_0) \right\} \tag{16}
\end{aligned}$$

where I_n is defined as:

$$I_n = \int_{m_Q^2}^{\infty} ds \frac{e^{m_Q^2/M^2 - s/M^2}}{s^n},$$

and other parameters and functions as well as the way of continuum subtraction are given in [14]. To shorten the equation, we have ignored the light quark masses as well as terms containing gluon condensates in the above equation, but we take into account their contributions in numerical analysis.

3 Numerical results

In this section, we present the numerical results of the sum rules for strong coupling constants of pseudoscalar mesons with sextet and antitriplet heavy baryons, which are obtained in the previous section. The main input parameters of LCSR are DA's for the pseudoscalar mesons which are given in [18–21]. The other input parameters entering to the sum rules are $\langle \bar{q}q \rangle = -(0.24 \pm 0.001)^3 \text{ GeV}^3$, $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [24], $f_{\pi} = 0.131 \text{ GeV}$, $f_K = 0.16 \text{ GeV}$ and $f_{\eta} = 0.13 \text{ GeV}$ [18].

The sum rules for the SSP, SAP and AAP coupling constants have three auxiliary parameters: Borel mass parameter M^2 , continuum threshold s_0 and the arbitrary parameter β which exists in the expression in the expression for the interpolating currents. Obviously, the result for any measurable physical quantity, being coupling constant in the present case, should be independent on them. Therefore, our primary goal is to find such regions of these parameters, where coupling constants exhibits no dependence.

The upper limit of M^2 is determined by requiring that the continuum and higher states contributions should be small compared to the total dispersion integral. The lower limit can be obtained from the condition that the condensate terms with highest dimensions contributes smaller compared to the sum of all terms. These two conditions leads to the working region, $15 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$ for the bottom baryons and $4 \text{ GeV}^2 \leq M^2 \leq 12 \text{ GeV}^2$ for the charmed ones. The continuum threshold is not totally arbitrary but it

depends on the energy of the first excited state with the same quantum numbers as the interpolating current. We choose it in the domain between $s_0 = (m_B + 0.5)^2 \text{ GeV}^2$ and $s_0 = (m_B + 1)^2 \text{ GeV}^2$. As an example, let us consider the $\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0$ transition. In Fig. (2), the dependence of the strong coupling constant for the $\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0$ transition on M^2 is considered at different fixed values of β and a fixed value of s_0 . We observe from this figure that the coupling constant has a good stability in the “working region” of M^2 . In Fig. (3), we present the dependence of the strong coupling constant for the $\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0$ transition on $\cos \theta$ at several fixed values of s_0 and at $M^2 = 22.5 \text{ GeV}^2$, where the angle θ is determined from $\beta = \tan \theta$. From this figure, we see that the dependence of the coupling constant on s_0 diminishes when the higher values of the continuum threshold are chosen from the considered working region. From this figure, we also observe that the strong coupling constant for the $\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0$ decay becomes very large near the end points ($\cos \theta = \pm 1$) and have zeros at some finite values of the $\cos \theta$. This behavior can be explained as follows. From Eq. (15) we see that the coupling constant is proportional to $\frac{1}{\lambda_1^{(i)} \lambda_2^{(i)}} \Pi_1^{(i)}$. In general, zero's of the nominator and denominator does not coincide since the OPE is truncated. In other words, calculations are not exact. For this reason, these points and any region between them are not reliable regions for determination of physical quantities and suitable regions for $\cos \theta$ should be far from these regions. It follows from Fig. (3) that in the region, $-0.5 \leq \cos \theta \leq +0.3$, the coupling constant seems to be insensitive to the variation of $\cos \theta$. Here, we should also stress that our numerical results lead to the working region, $-0.6 \leq \cos \theta \leq +0.5$ common for masses of all heavy spin 1/2 baryons, which includes the working region of $\cos \theta$ for the coupling constant. This region lie also inside the more wide interval of $\cos \theta$ obtained from analysis of the masses of the non strange heavy baryons in [10, 25, 26]. In general, the working region of $\cos \theta$ for masses and coupling constants can be different, but in some cases as occur in our problem these regions coincide.

Similar analysis for the strong coupling constants of the light pseudoscalar mesons with sextet and antitriplet heavy baryons are performed and the results are presented in Tables (2), (3) and (4). In these Tables we also present the predictions for the coupling constants coming from the Ioffe currents when $\beta = -1$. The errors in the values of the coupling constants presented in the Tables (2), (3) and (4) include uncertainties coming from the variations of the s_0 , β and M^2 as well as those coming from the other input parameters.

g channel	Bottom Baryons		g channel	Charmed Baryons	
	General current	Ioffe current		General current	Ioffe current
$g^{\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0}$	9.0 ± 3.0	7.3 ± 2.6	$g^{\Xi_c'^+ \rightarrow \Xi_c'^+ \pi^0}$	4.0 ± 1.4	3.0 ± 1.1
$g^{\Sigma_b^0 \rightarrow \Sigma_b^- \pi^+}$	17.0 ± 6.1	13.0 ± 4.5	$g^{\Sigma_c^+ \rightarrow \Sigma_c^0 \pi^+}$	8.0 ± 2.8	4.1 ± 1.5
$g^{\Xi_b'^0 \rightarrow \Sigma_b^+ K^-}$	19.0 ± 6.7	10.0 ± 3.6	$g^{\Xi_c'^+ \rightarrow \Sigma_c^{++} K^-}$	9.0 ± 3.4	3.0 ± 1.0
$g^{\Omega_b^- \rightarrow \Xi_b'^0 K^-}$	21.0 ± 6.8	12.3 ± 4.4	$g^{\Omega_c^0 \rightarrow \Xi_c'^+ K^-}$	9.0 ± 3.4	5.6 ± 1.9
$g^{\Sigma_b^+ \rightarrow \Sigma_b^+ \eta_1}$	12.5 ± 4.4	8.7 ± 3.1	$g^{\Sigma_c^{++} \rightarrow \Sigma_c^{++} \eta_1}$	6.0 ± 2.2	2.8 ± 1.0
$g^{\Xi_b'^0 \rightarrow \Xi_b'^0 \eta_1}$	5.3 ± 1.9	3.6 ± 1.3	$g^{\Xi_c'^+ \rightarrow \Xi_c'^+ \eta_1}$	2.6 ± 0.9	0.7 ± 0.2
$g^{\Omega_b^- \rightarrow \Omega_b^- \eta_1}$	26.0 ± 7.4	20.0 ± 5.5	$g^{\Omega_c^0 \rightarrow \Omega_c^0 \eta_1}$	11.0 ± 3.8	9.3 ± 3.4

Table 2: The values of the strong coupling constants g for the transitions among the sextet and sextet heavy baryons with pseudoscalar mesons.

g channel	Bottom Baryons		g channel	Charmed Baryons	
	General current	Ioffe current		General current	Ioffe current
$g^{\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0}$	7.5 ± 2.6	6.1 ± 2.2	$g^{\Xi_c'^+ \rightarrow \Xi_c'^+ \pi^0}$	3.1 ± 1.1	2.0 ± 0.7
$g^{\Sigma_b^- \rightarrow \Lambda_b^0 \pi^-}$	15.0 ± 4.9	11.5 ± 3.9	$g^{\Sigma_c^0 \rightarrow \Lambda_c^+ \pi^-}$	6.5 ± 2.4	5.6 ± 1.8
$g^{\Sigma_b^0 \rightarrow \Xi_b^0 \bar{K}^0}$	11.5 ± 3.9	8.9 ± 3.1	$g^{\Sigma_c^+ \rightarrow \Xi_c^+ \bar{K}^0}$	5.0 ± 1.7	3.7 ± 1.3
$g^{\Omega_b^- \rightarrow \Xi_b^- \bar{K}^0}$	17.0 ± 4.5	13.5 ± 4.8	$g^{\Omega_c^0 \rightarrow \Xi_c^0 \bar{K}^0}$	6.5 ± 2.3	3.0 ± 1.1
$g^{\Xi_b'^0 \rightarrow \Xi_b^- K^+}$	12.0 ± 4.3	9.8 ± 3.5	$g^{\Xi_c'^+ \rightarrow \Xi_c^0 K^+}$	4.5 ± 1.6	2.1 ± 0.8
$g^{\Xi_b'^0 \rightarrow \Xi_b^0 \eta_1}$	16.0 ± 5.6	12.0 ± 4.3	$g^{\Xi_c'^+ \rightarrow \Xi_c^+ \eta_1}$	6.7 ± 2.4	4.3 ± 1.5

Table 3: The values of the strong coupling constants g for the transitions among the sextet and antitriplet heavy baryons with pseudoscalar mesons.

We see from these Tables that, there is substantial difference between the predictions of the general current and the Ioffe current, especially for the strong coupling constants of the antitriplet–antitriplet heavy baryons with pseudoscalar mesons, which can be explained as follows. As a result of the analysis of the dependence of the coupling constants on $\cos \theta$ we see that the value $\beta = -1$ belongs to the unstable region. Therefore, a prediction at this point of β is not reliable.

Finally, we compare our results with those existing in literature. In various works, the coupling constant $\Sigma_c \rightarrow \Lambda_c \pi$ is estimated to be

$$g^{\Sigma_c \rightarrow \Lambda_c \pi} = \begin{cases} 8.88, [27] \text{ (relativistic three-quark model) }, \\ 6.82, [28] \text{ (light-front quark model) }, \\ 10.8 \pm 2.2, [9] \text{ (LCSR) }, \\ 6.5 \pm 2.4, \text{ (our result) (LCSR) }. \end{cases}$$

g channel	Bottom Baryons		g channel	Charmed Baryons	
	General current	Ioffe current		General current	Ioffe current
$g_{\Xi_b^0 \rightarrow \Xi_b^0 \pi^0}$	1.0 ± 0.3	4.0 ± 1.4	$g_{\Xi_c^+ \rightarrow \Xi_c^+ \pi^0}$	0.70 ± 0.22	2.7 ± 0.9
$g_{\Xi_b^- \rightarrow \Lambda_b^0 K^-}$	1.5 ± 0.5	5.2 ± 1.8	$g_{\Xi_c^0 \rightarrow \Lambda_c^+ K^-}$	0.9 ± 0.3	2.2 ± 0.7
$g_{\Xi_b^0 \rightarrow \Xi_b^0 \eta_1}$	0.6 ± 0.2	2.9 ± 1.0	$g_{\Xi_c^+ \rightarrow \Xi_c^+ \eta_1}$	0.07 ± 0.02	0.26 ± 0.08
$g_{\Lambda_b^0 \rightarrow \Lambda_b^0 \eta_1}$	1.0 ± 0.3	4.0 ± 1.1	$g_{\Lambda_c^+ \rightarrow \Lambda_c^+ \eta_1}$	0.75 ± 0.24	1.9 ± 0.66

Table 4: The values of the strong coupling constants g for the transitions among the antitriplet and antitriplet heavy baryons with pseudoscalar mesons.

We see that, within errors our result is close to the results of [9, 27, 28]. The coupling constant for the $\Xi_Q \Xi_Q \pi$ transition LCSR is estimated to have the values $g^{\Xi_c \rightarrow \Xi_c \pi} = 1.0 \pm 0.5$ and $g^{\Xi_b \rightarrow \Xi_b \pi} = 1.6 \pm 0.4$, which are slightly larger compared to our predictions. Finally, the coupling constant $g^{\Sigma_c \rightarrow \Sigma_c \pi}$ is calculated in [9] and it is obtained that $g^{\Sigma_c \rightarrow \Sigma_c \pi} = -8.0 \pm 1.7$, which is in quite a good agreement with our prediction.

In conclusion, the strong coupling constants of light pseudoscalar mesons with sextet and antitriplet heavy baryons are studied within LCSR. It is shown that, all coupling constants for the sextet–sextet, sextet–antitriplet and antitriplet–antitriplet transitions are described by only one invariant function in each class.

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Appendix A :

Here in this appendix, we present the expressions of the correlation functions in terms of invariant function $\Pi_1^{(i)}$ involving π , K and η mesons.

- Correlation functions describing pseudoscalar mesons with sextet–sextet baryons.

$$\begin{aligned}
\frac{1}{\sqrt{2}} \Pi^{\Sigma_b^+ \rightarrow \Sigma_b^0 \pi^+} &= \Pi^{\Xi_b'^0 \rightarrow \Sigma_b^0 \bar{K}^0} = \Pi^{\Sigma_b^0 \rightarrow \Xi_b'^0 K^0} = \Pi_1^{(1)}(d, u, b) , \\
\Pi^{\Xi_b'^0 \rightarrow \Xi_b'^- \pi^+} &= \Pi_1^{(1)}(d, s, b) , \\
\frac{1}{\sqrt{2}} \Pi^{\Sigma_b^- \rightarrow \Sigma_b^0 \pi^-} &= \Pi^{\Xi_b'^- \rightarrow \Sigma_b^0 K^-} = \Pi^{\Sigma_b^0 \rightarrow \Xi_b'^- K^+} = \Pi_1^{(1)}(u, d, b) , \\
\Pi^{\Xi_b'^- \rightarrow \Xi_b'^0 \pi^-} &= \Pi_1^{(1)}(u, s, b) , \\
\frac{1}{\sqrt{2}} \Pi^{\Xi_b'^0 \rightarrow \Sigma_b^+ K^-} &= \frac{1}{\sqrt{2}} \Pi^{\Sigma_b^+ \rightarrow \Xi_b'^0 K^+} = \frac{\sqrt{6}}{2} \Pi^{\Sigma_b^+ \rightarrow \Sigma_b^+ \eta_1} = \Pi_1^{(1)}(u, u, b) ,
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\sqrt{2}}\Pi^{\Omega_b^- \rightarrow \Xi_b'^0 K^-} &= \frac{1}{\sqrt{2}}\Pi^{\Xi_b'^0 \rightarrow \Omega_b^- K^+} = \frac{1}{\sqrt{2}}\Pi^{\Omega_b^- \rightarrow \Xi_b'^- \bar{K}^0} = \frac{1}{\sqrt{2}}\Pi^{\Xi_b'^- \rightarrow \Omega_b^- K^0} = -\frac{\sqrt{6}}{4}\Pi^{\Omega_b^- \rightarrow \Omega_b^- \eta_1} \Pi_1^{(1)}(s, s, b) , \\
\frac{1}{\sqrt{2}}\Pi^{\Xi_b'^- \rightarrow \Sigma_b^- \bar{K}^0} &= \frac{1}{\sqrt{2}}\Pi^{\Sigma_b^- \rightarrow \Xi_b'^- K^0} = \frac{\sqrt{6}}{2}\Pi^{\Sigma_b^- \rightarrow \Sigma_b^- \eta_1} = \Pi_1^{(1)}(d, d, b) , \\
\Pi^{\Xi_b'^0 \rightarrow \Xi_b'^0 \eta_1} &= \frac{1}{\sqrt{6}} \left[\Pi_1^{(1)}(u, s, b) - 2\Pi_1^{(1)}(s, u, b) \right] , \\
\Pi^{\Xi_b'^- \rightarrow \Xi_b'^- \eta_1} &= \frac{1}{\sqrt{6}} \left[\Pi_1^{(1)}(d, s, b) - 2\Pi_1^{(1)}(s, d, b) \right] ,
\end{aligned} \tag{A.1}$$

- Correlation functions responsible for the transitions of the sextet–antitriplet baryons.

$$\begin{aligned}
\sqrt{2}\Pi^{\Xi_b'^0 \rightarrow \Xi_b^0 \pi^0} &= \Pi^{\Xi_b'^0 \rightarrow \Xi_b^- \pi^+} = \Pi^{\Xi_b'^- \rightarrow \Xi_b^0 K^-} = \Pi_1^{(2)}(u, s, b) , \\
-\sqrt{2}\Pi^{\Xi_b'^- \rightarrow \Xi_b^- \pi^0} &= \Pi^{\Xi_b'^- \rightarrow \Xi_b^0 \pi^-} = \Pi_1^{(2)}(d, s, b) , \\
\Pi^{\Sigma_b^0 \rightarrow \Lambda_b^0 \pi^0} &= \frac{1}{\sqrt{2}} \left[\Pi_1^{(2)}(u, d, b) + \Pi_1^{(2)}(d, u, b) \right] , \\
\Pi^{\Sigma_b^- \rightarrow \Lambda_b^0 \pi^-} &= -\Pi^{\Sigma_b^0 \rightarrow \Xi_b^- K^+} = \Pi_1^{(2)}(u, d, b) , \\
-\frac{1}{\sqrt{2}}\Pi^{\Sigma_b^+ \rightarrow \Lambda_b^0 \pi^+} &= -\Pi^{\Sigma_b^0 \rightarrow \Xi_b^0 \bar{K}^0} = -\Pi^{\Xi_b'^0 \rightarrow \Lambda_b^0 \bar{K}^0} = \Pi^{\Xi_b'^0 \rightarrow \Xi_b^- K^+} = \Pi_1^{(2)}(d, u, b) , \\
-\frac{1}{\sqrt{2}}\Pi^{\Sigma_b^- \rightarrow \Xi_b^- \bar{K}^0} &= -\frac{1}{\sqrt{2}}\Pi^{\Sigma_b^- \rightarrow \Lambda_b^0 K^-} = \Pi_1^{(2)}(d, d, b) , \\
\frac{1}{\sqrt{2}}\Pi^{\Omega_b^- \rightarrow \Xi_b^- \bar{K}^0} &= \frac{1}{\sqrt{2}}\Pi^{\Omega_b^- \rightarrow \Xi_b^0 K^-} = \Pi_1^{(2)}(s, s, b) , \\
\Pi^{\Sigma_b^+ \rightarrow \Lambda_b^0 K^+} &= -\sqrt{2}\Pi_1^{(2)}(u, u, b) , \\
\Pi^{\Xi_b'^0 \rightarrow \Xi_b^0 \eta_1} &= \frac{1}{\sqrt{6}} \left[\Pi_1^{(2)}(u, s, b) + 2\Pi_1^{(2)}(s, u, b) \right] , \\
\Pi^{\Xi_b'^- \rightarrow \Xi_b^- \eta_1} &= \frac{1}{\sqrt{6}} \left[\Pi_1^{(2)}(d, s, b) + 2\Pi_1^{(2)}(s, d, b) \right] , \\
\Pi^{\Sigma_b^0 \rightarrow \Lambda_b^0 \eta_1} &= \frac{1}{\sqrt{6}} \left[\Pi_1^{(2)}(u, d, b) - \Pi_1^{(2)}(d, u, b) \right] .
\end{aligned}$$

- Correlation functions appearing in the antitriplet–antitriplet pseudoscalar meson transitions.

$$\begin{aligned}
\sqrt{2}\Pi^{\Xi_b^0 \rightarrow \Xi_b^0 \pi^0} &= \Pi^{\Xi_b^0 \rightarrow \Xi_b^- \pi^+} = \Pi_1^{(3)}(u, s, b) , \\
-\sqrt{2}\Pi^{\Xi_b^- \rightarrow \Xi_b^- \pi^0} &= \Pi^{\Xi_b^- \rightarrow \Xi_b^0 \pi^-} = \Pi_1^{(3)}(d, s, b) , \\
\Pi^{\Lambda_b^0 \rightarrow \Lambda_b^0 \pi^0} &= \frac{1}{\sqrt{2}} \left[\Pi_1^{(3)}(u, d, b) - \Pi_1^{(3)}(d, u, b) \right] , \\
\Pi^{\Xi_b^0 \rightarrow \Lambda_b^0 \bar{K}^0} &= \Pi_1^{(3)}(u, u, b) , \\
\Pi^{\Xi_b^- \rightarrow \Lambda_b^0 K^-} &= -\Pi_1^{(3)}(u, d, b) ,
\end{aligned}$$

$$\begin{aligned}
\Pi^{\Xi_b^0 \rightarrow \Xi_b^0 \eta_1} &= \frac{1}{\sqrt{6}} \left[\Pi_1^{(3)}(u, s, b) - 2\Pi_1^{(3)}(s, u, b) \right] , \\
\Pi^{\Xi_b^- \rightarrow \Xi_b^- \eta_1} &= \frac{1}{\sqrt{6}} \left[\Pi_1^{(3)}(d, s, b) - 2\Pi_1^{(3)}(s, d, b) \right] , \\
\Pi^{\Lambda_b^0 \rightarrow \Lambda_b^0 \eta_1} &= \frac{1}{\sqrt{6}} \left[\Pi_1^{(3)}(d, u, b) + \Pi_1^{(3)}(u, d, b) \right] .
\end{aligned}$$

The expressions for the charmed baryons can easily be obtained by making the replacement $b \rightarrow c$ and adding to charge of each baryon a positive unit charge.

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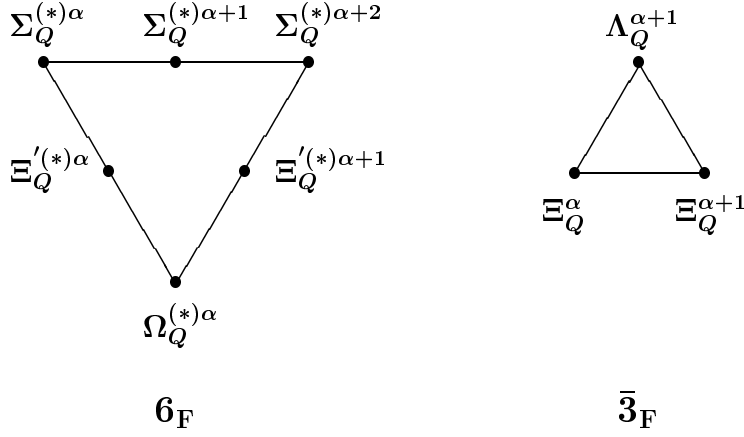


Figure 1: Sextet (6_F) and antitriplet ($\bar{3}_F$) representations of heavy baryons. Here α , $\alpha + 1$, $\alpha + 2$ determine the charges of baryons ($\alpha = -1$ or 0), and $(*)$ denote $J^P = \frac{3}{2}^+$ states.

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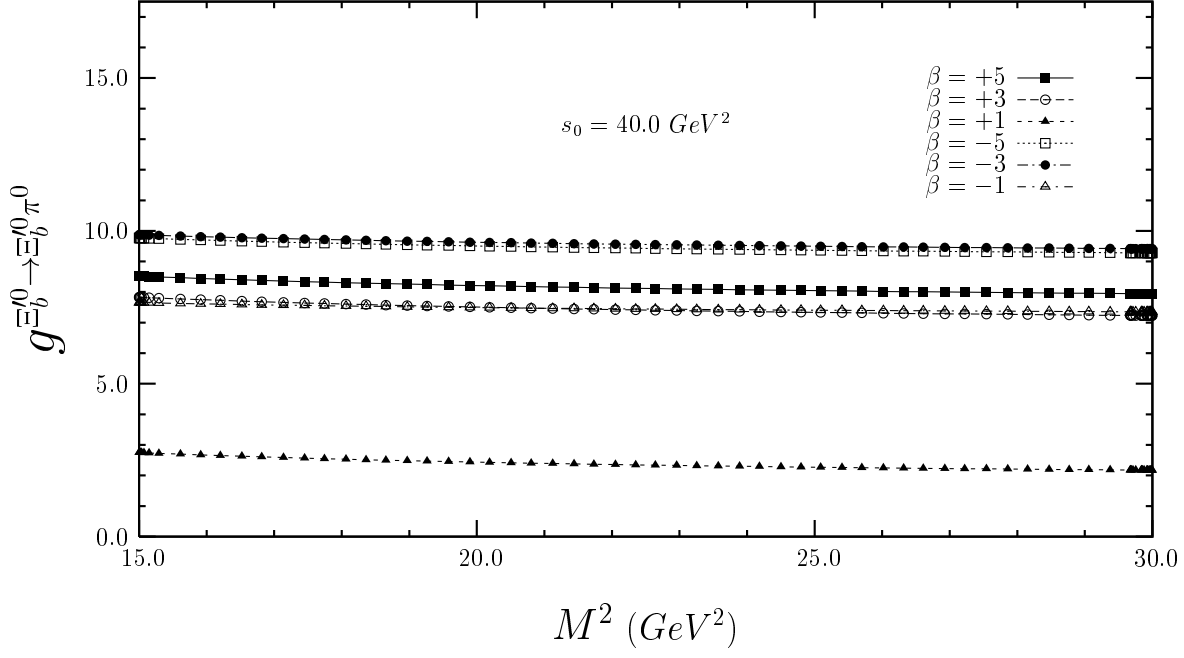


Figure 2: The dependence of the strong coupling constant for the $\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0$ transition at several different fixed values of β , and at $s_0 = 40.0 \text{ GeV}^2$.

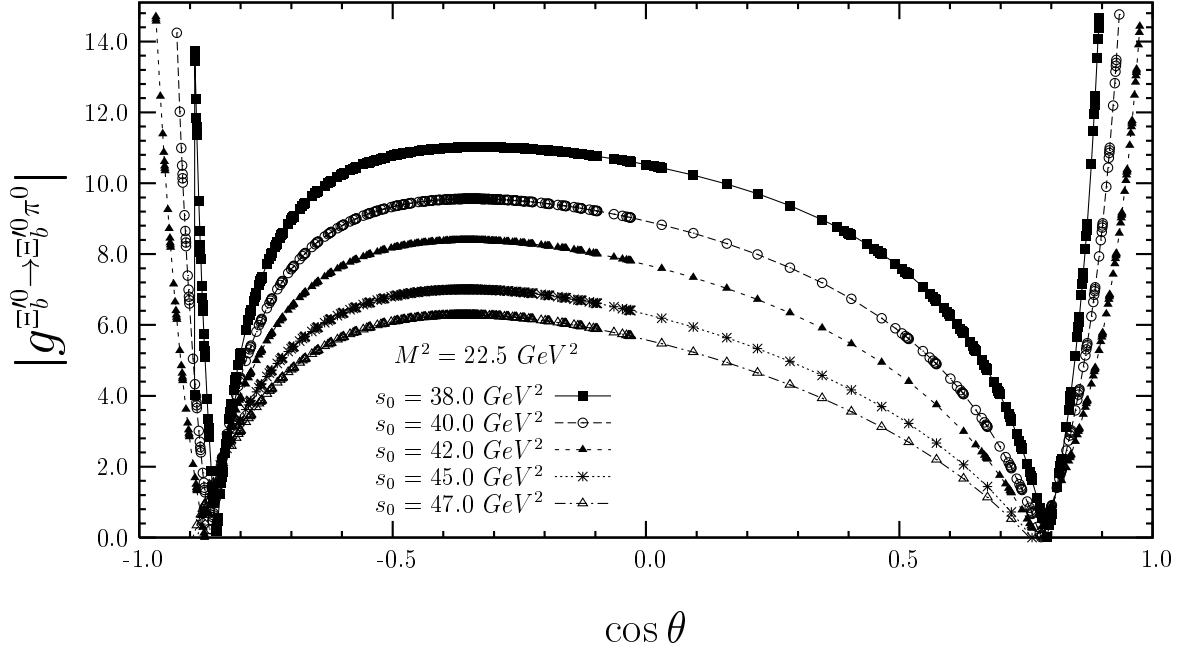


Figure 3: The dependence of the strong coupling constant for the $\Xi_b'^0 \rightarrow \Xi_b'^0 \pi^0$ transition on $\cos \theta$ at several different fixed values of s_0 , and at $M^2 = 22.5 \text{ GeV}^2$.